1 Independence, Baron p. 40

Events $A$ and $B$ are independent. Show, intuitively, and mathematically, that

(a) Their complements are also independent.

(b) If they are disjoint, then $P(A) = 0$ or $P(B) = 0$.

(c) If they are exhaustive (i.e. $A \cup B = \Omega$), then $P(A) = 1$ or $P(B) = 1$.

(d) If $P(B) \neq 0$, then $P(A|B) = P(A)$. Also, show that if $P(B) \neq 0$ and $P(A|B) = P(A)$, then $A$ and $B$ are independent.

2 Bayes’ Theorem

(a) Suppose that a barrel contains many small plastic eggs. Some eggs are painted red and some are painted blue. 40% of the eggs in the bin contain pearls, and 60% contain nothing. 30% of eggs containing pearls are painted blue, and 10% of eggs containing nothing are painted blue. What is the probability that a blue egg contains a pearl?

(b) 1% of women at age forty who participate in routine screening have breast cancer. 80% of women with breast cancer will get positive mammographies. 9.6% of women without breast cancer will also get positive mammographies. A woman in this age group had a positive mammography in a routine screening. What is the probability that she actually has breast cancer?

3 Debugging with Probabilities

Consider the following program segment:

```plaintext
if B then
  while B1 do S
else
  while B2 do S
```

Assume that $P(B = \text{true}) = p$, $P(B_1 = \text{true}) = 2/5$, and $P(B_2 = \text{true}) = 3/5$. $S$ prints the message 'good day' in each iteration. Past evaluations have shown that the probability for exactly two 'good day' messages is $3/25$. Use a tree diagram to determine the value of $p$ (You will not be able to draw the whole tree, since it is infinitely large. Just draw the part you need to solve the problem.)

4 Random Variables

Describe in detail two real life examples each for a situation, where a discrete random variable or a continuous random variable would be involved. For each situation, define the random variable and identify its image.
5 Missile Protection System

A missile protection system consists of \( n \) radar sets operating independently, each with probability 0.9 of detecting a missile entering a zone that is covered by all of the units.

a) Find a nice expression for the probability that at least one unit detects the missile. (Tip: what is the probability, that a missile is not detected by any of the sets? - what has this probability to do with the above?)

b) How large must \( n \) be if we require that the probability of detecting a missile that enters the zone be 0.999 ? 0.999999 ?

c) If \( n = 5 \) and a missile enters the zone, what is the probability that exactly 4 sets detect the missile? At least one set?